

Non-Linearity in Factor Returns

EDHEC SPEAKER SERIES

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Basics of Linear Factor Return Model

- The expected return (r_i) for stock i is expressed as a function of factor exposures (c_i)

$$r_i = c_1 s_{1,i} + c_2 s_{2,i} + c_3 s_{3,i} + c_4 s_{4,i} + c_5 s_{5,i} + \varepsilon_i$$

- Exposure are typically factors such as earnings yield (i.e., inverse trailing P/E), log price momentum, negative log market capitalization, negative trailing 36-month market beta, and prior-year gross-margin
- Exposure are usually expressed in Z score (i.e. units of standard deviation)
- Return associated with exposure ($c_1, c_2 \dots$) usually based on historical analysis

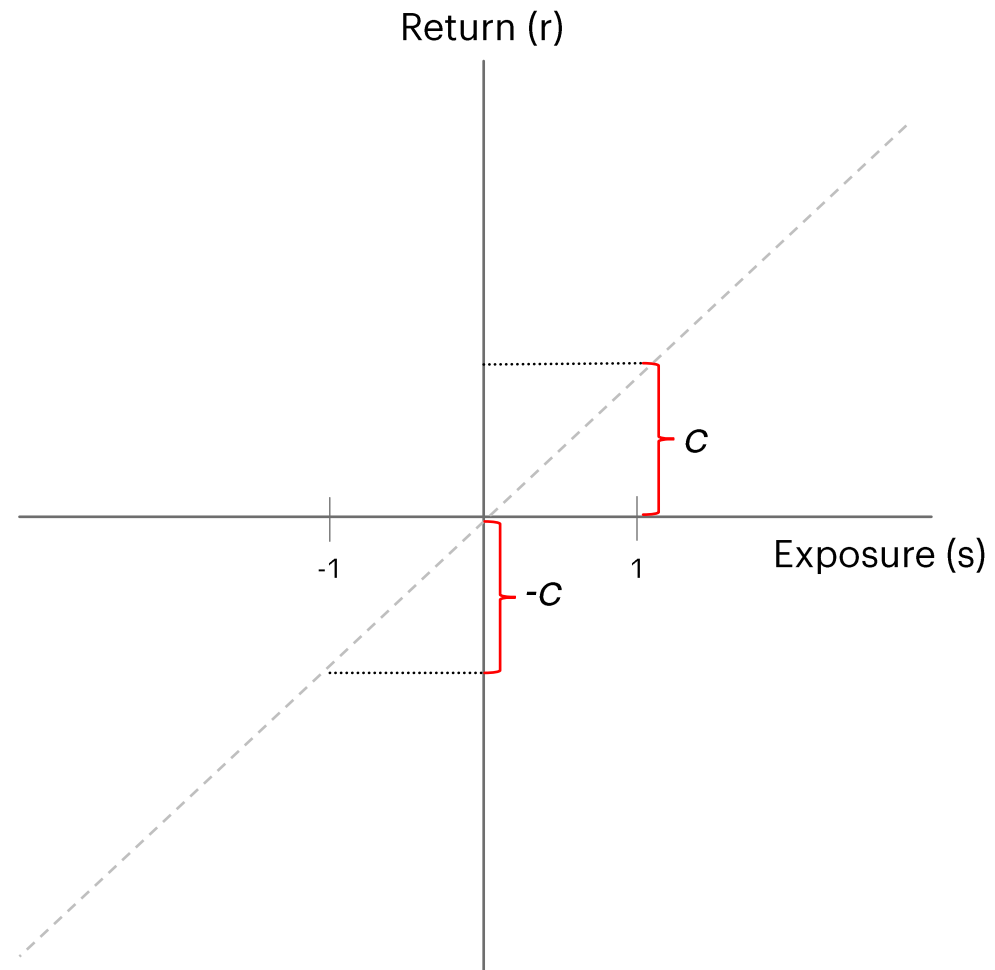
Implications

Expected Returns (or Attribution)

- A positive active weight of 2% in a stock of +1 is equivalent to -2% underweight of stock with exposure of -1
- A positive active weight of 2% in a stock with +1 is same as 1% to a stock with exposure of +2.
- Weighted average portfolio score can be used to form return expectation

Risk

- Tracking Error (or Risk) is linearly related to squared exposure value
- Average exposure is sufficient to capture risk profile of portfolio





Findings

- Significant non-linearities in return premia associated with factors in US equity market
 - Implications for portfolio management and attribution
 - Implications for active risk management
- Accounting for non-linearity can be significant source of value added
- Study is based on 1000 largest stocks in the US Market over the time period 1964-2023 using Small Size, Value, Low Beta, Momentum and Profitability



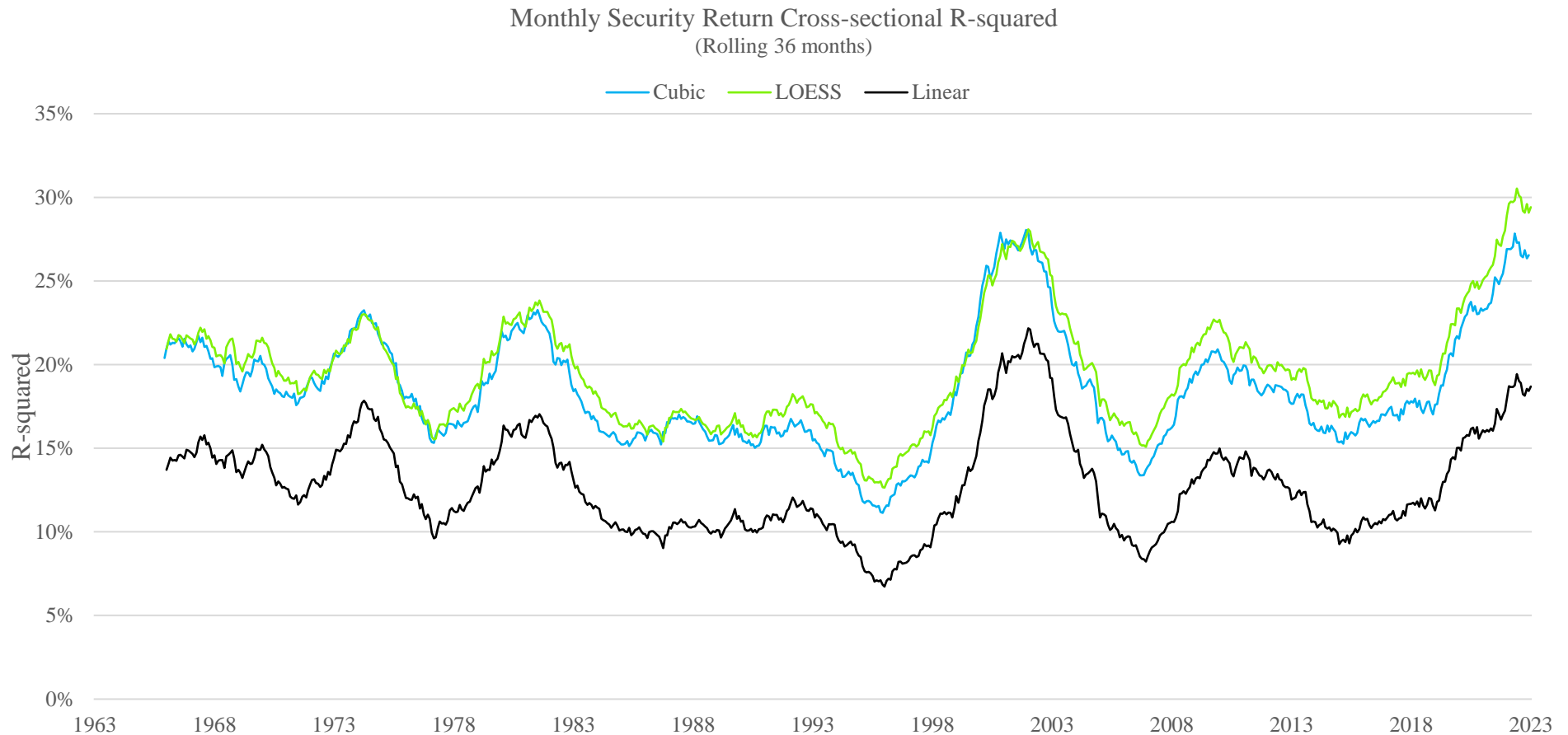
Basics of Non-Linear Factor Return Model

- The expected return (r_i) for stock i is expressed as an expansion of linear equation that includes orthogonalized squared and cubed characteristic scores for each factor

$$r_i = \lambda_1 s_i + \lambda_2 s_i^2 + \lambda_3 s_i^3 + \varepsilon_i$$

- Where s_i is the scored characteristic, s_i^2 is based on the squared characteristic, and s_i^3 is the cubed characteristic
- Squared and cubed terms are rescored after orthogonalization to a weighted mean of zero and a weighted variance of one
- $\lambda_1, \lambda_2, \lambda_3$ the slope coefficients can be equivalently estimated by three individual univariate regressions

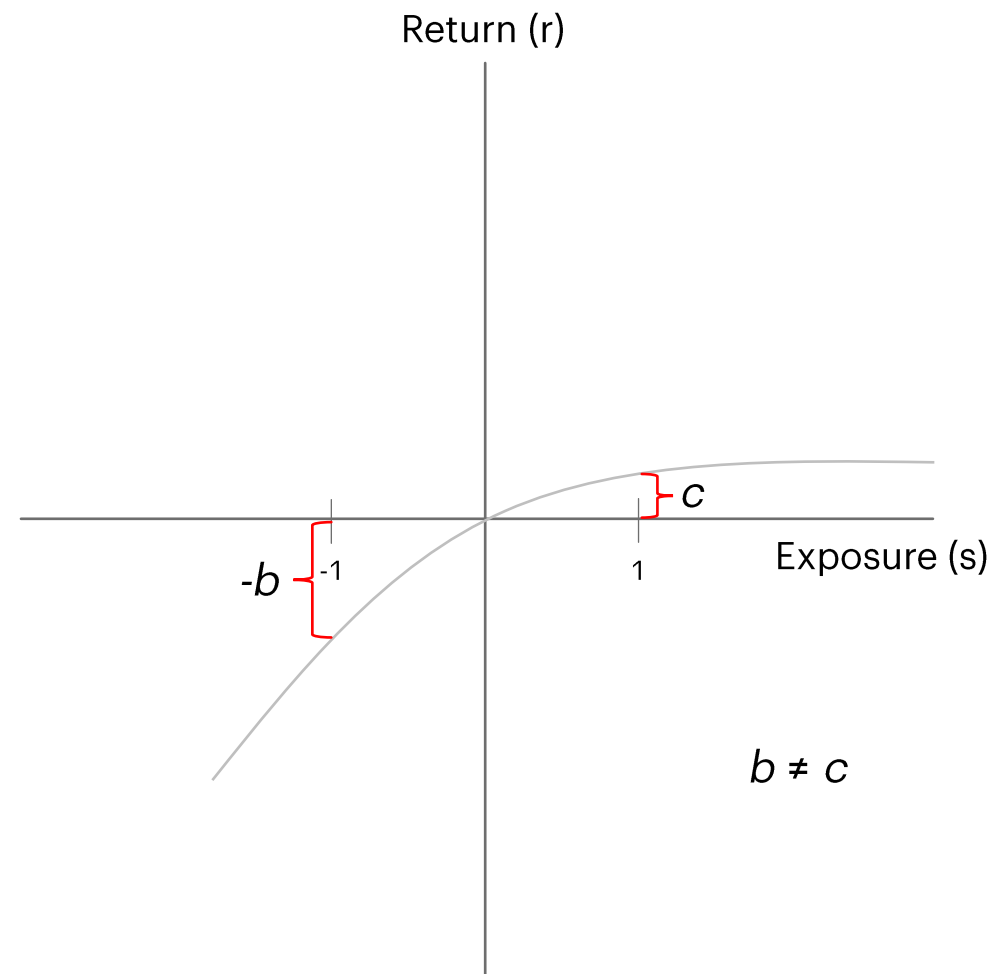
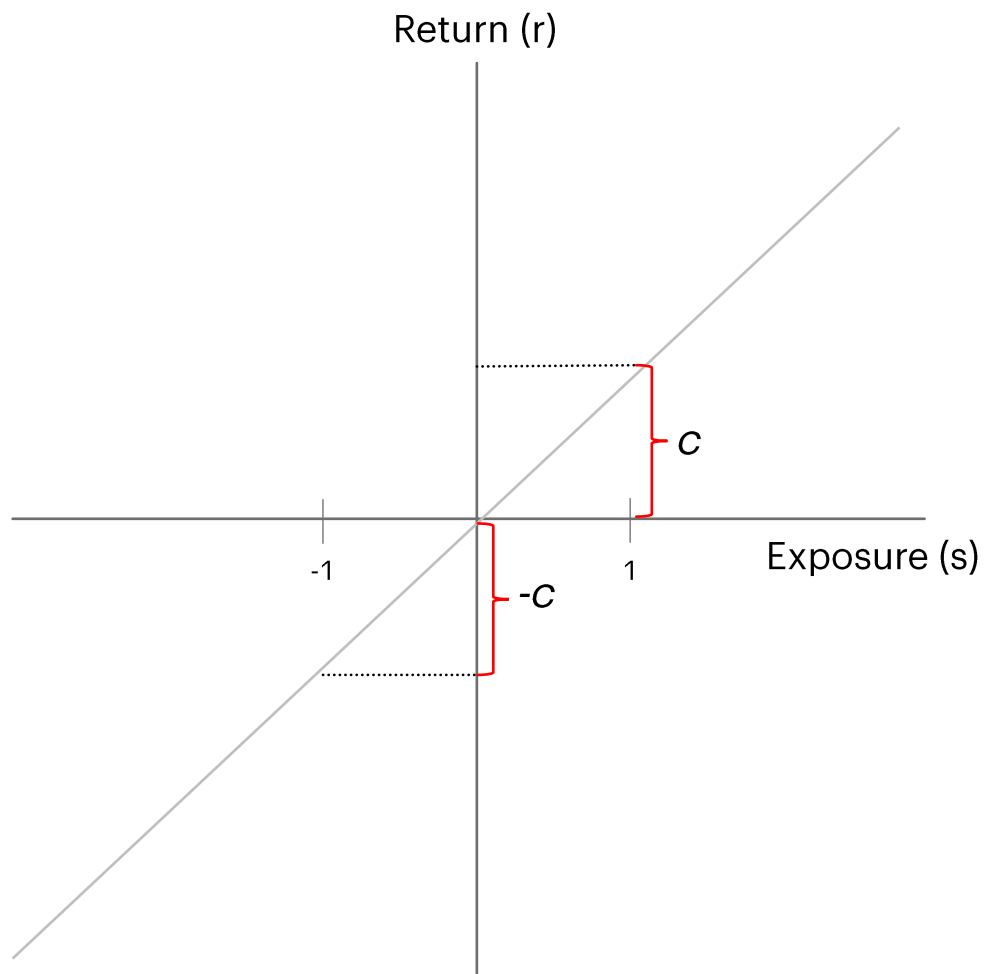
Benefits of Modeling Non-Linearity



LOESS (locally estimated scatterplot smoothing) allows for a non-polynomial shape in the regression curves that replace the terms in Equation 1. LOESS is computationally difficult to implement in our weighted-observation large-sample context.



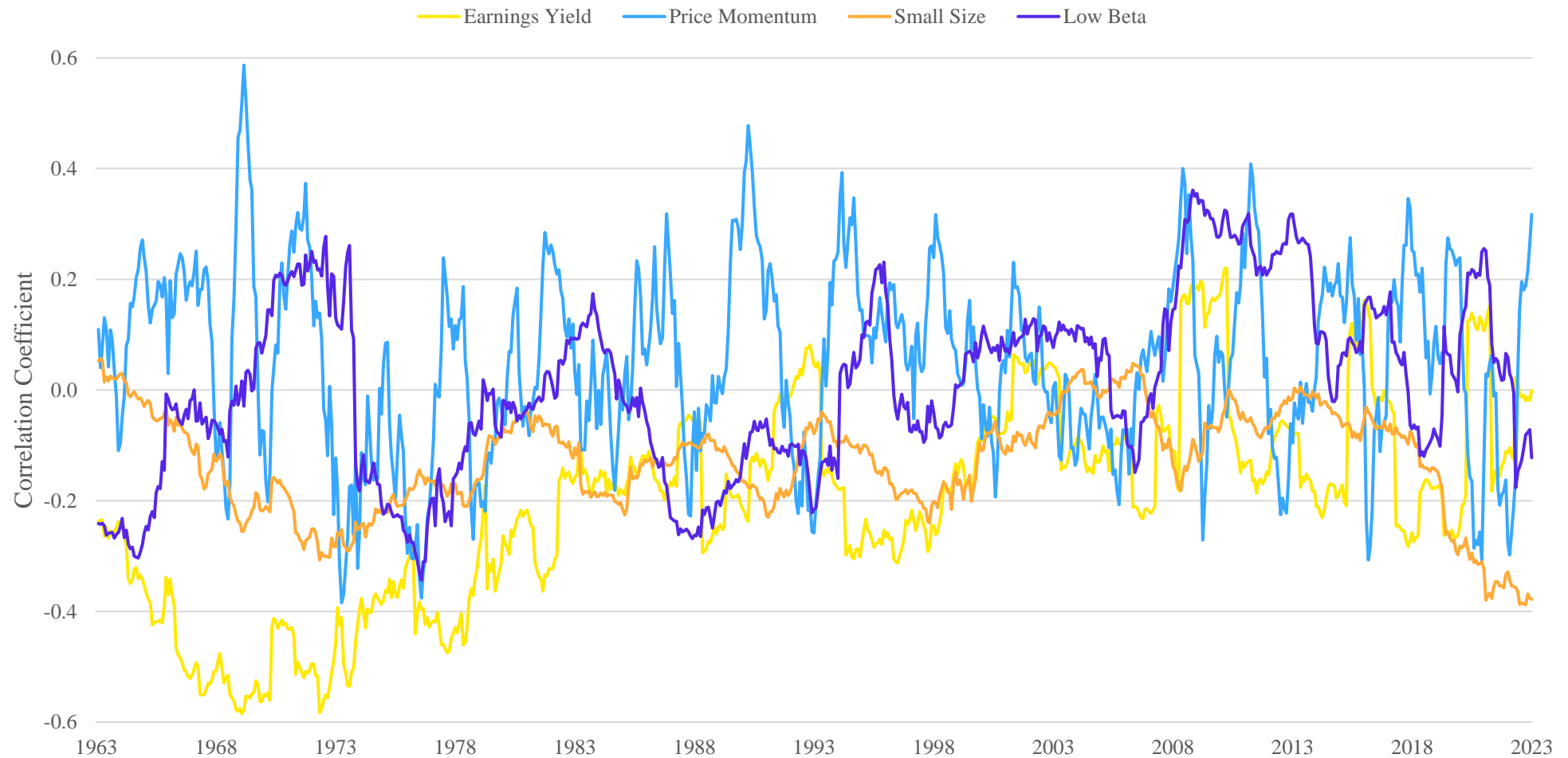
Linear vs. Non-Linear Approach





Importance of Purifying Factors

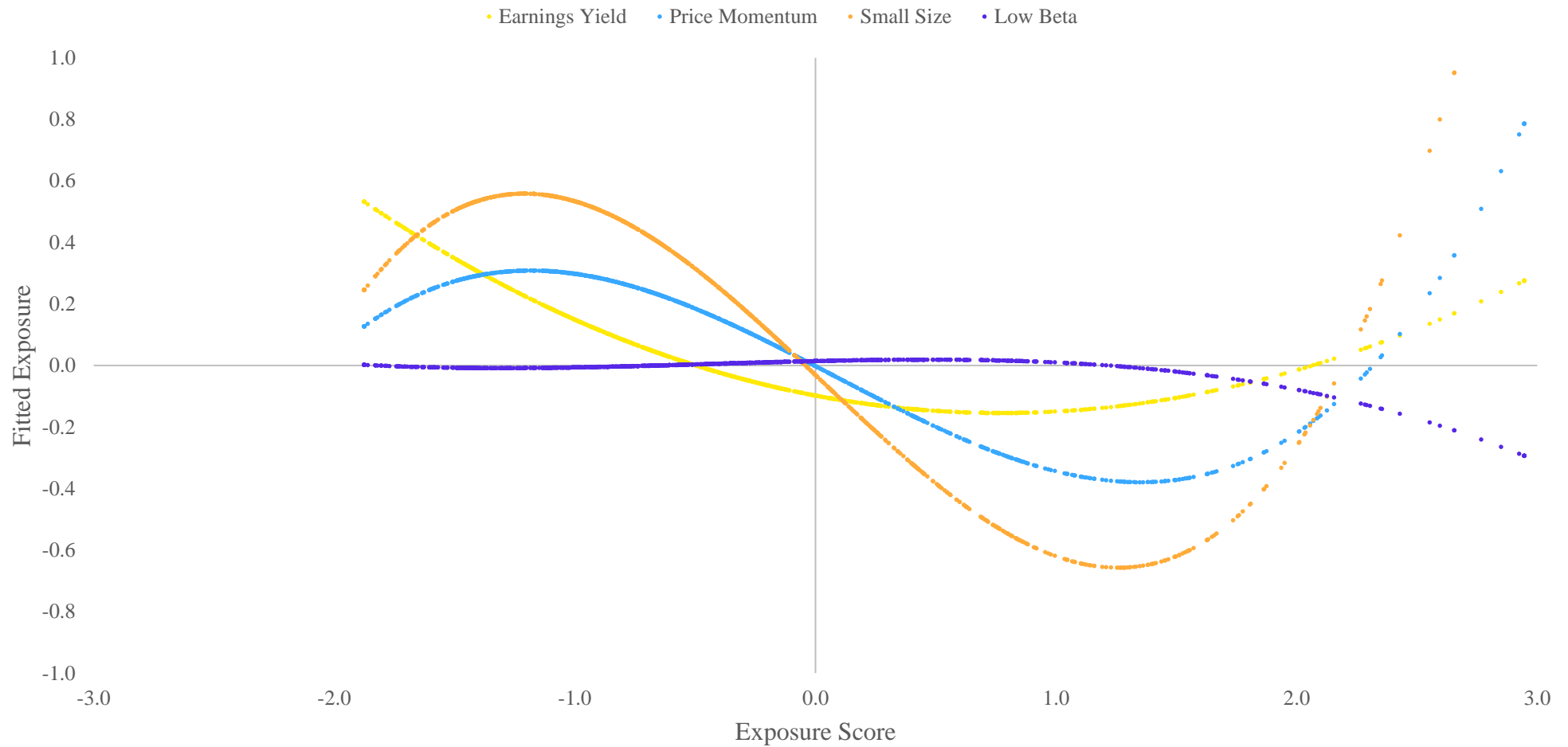
Gross Margin Exposure Linear Correlations Over Time





Non-Linear Purification

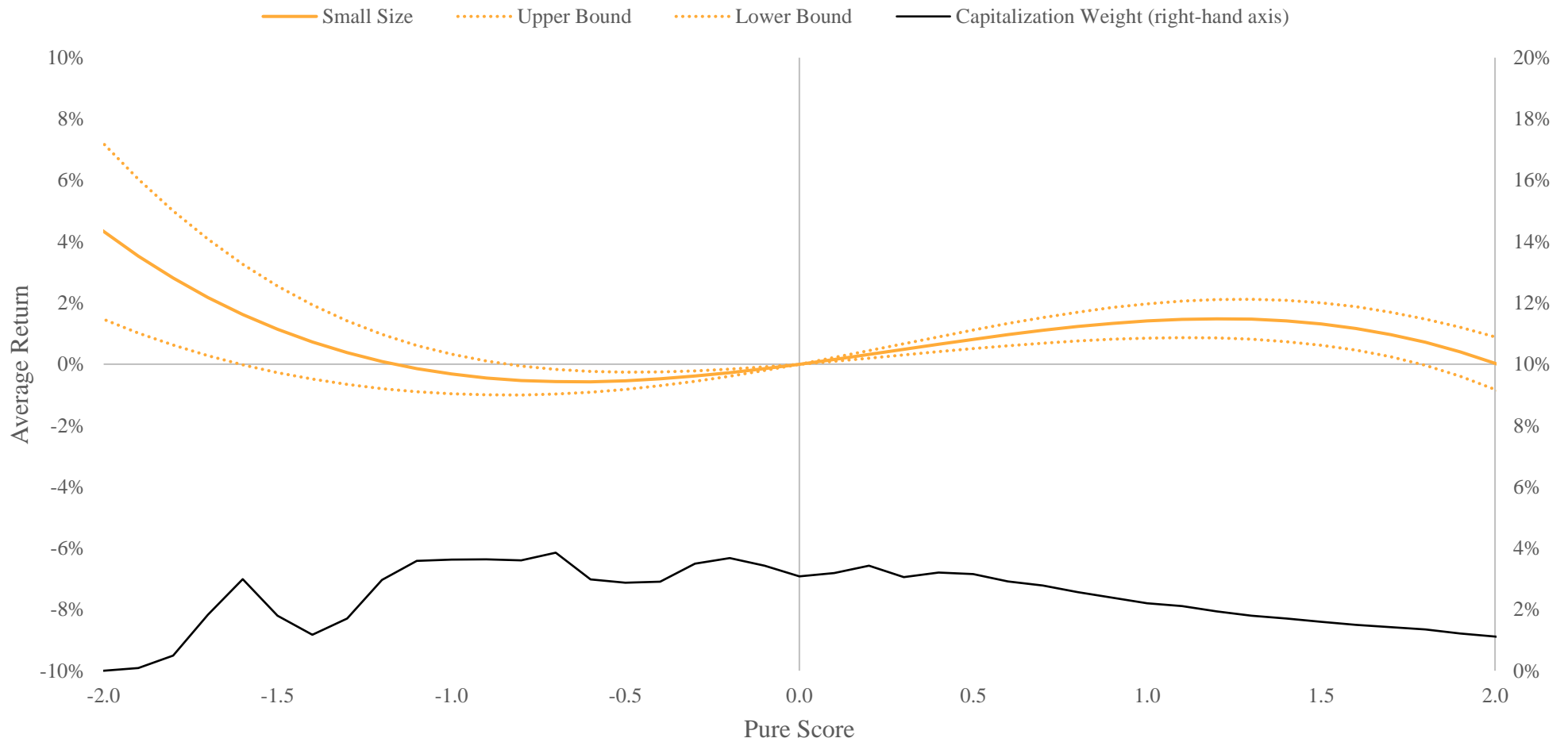
Gross Margin Non-Linear Relationships 2023





Return to Small Size

Non-Linear Average Return to Pure Score 2004 to 2023

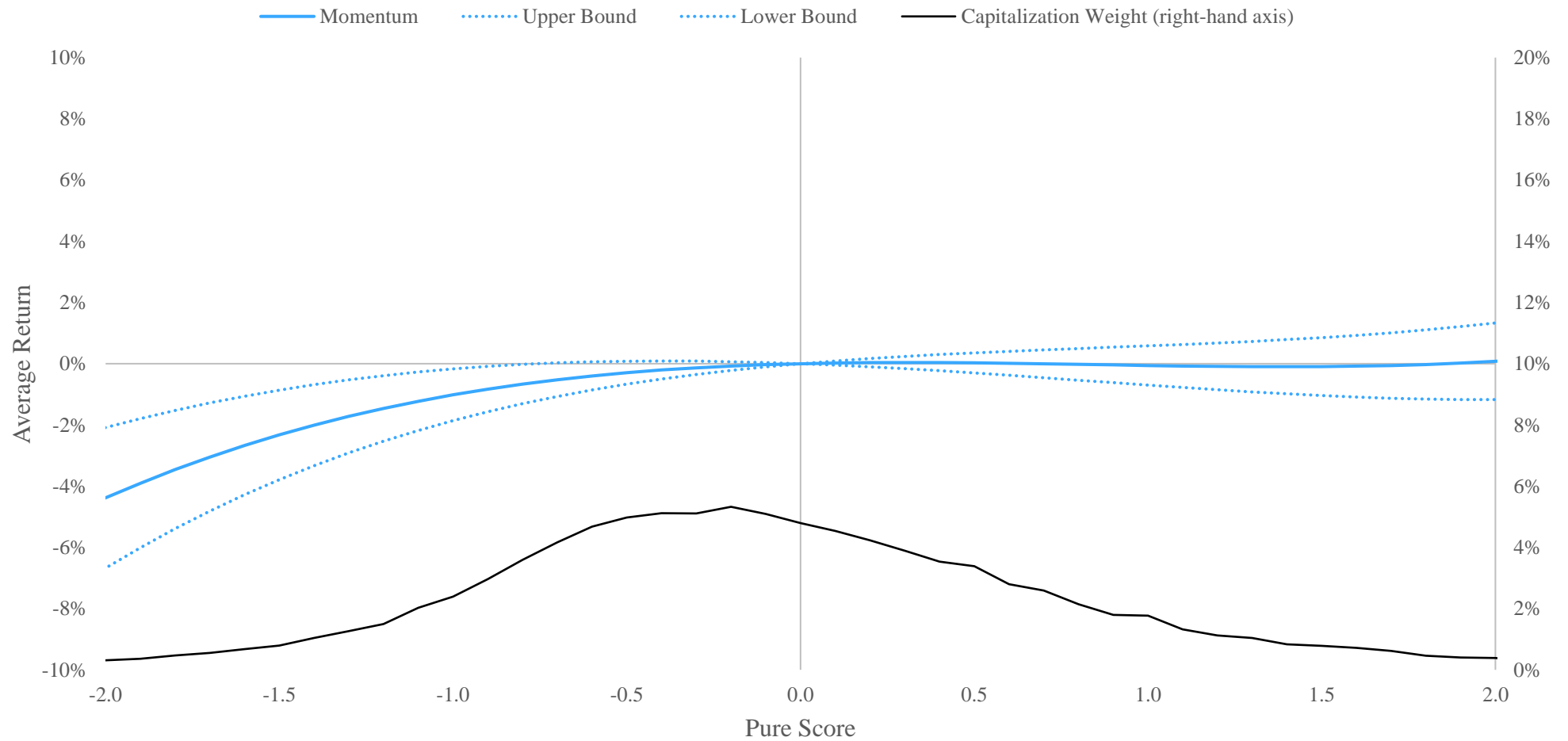


Note: Small Size is negative log capitalization at the beginning of the month



Return to Price Momentum

Non-Linear Average Return to Pure Score 2004 to 2023

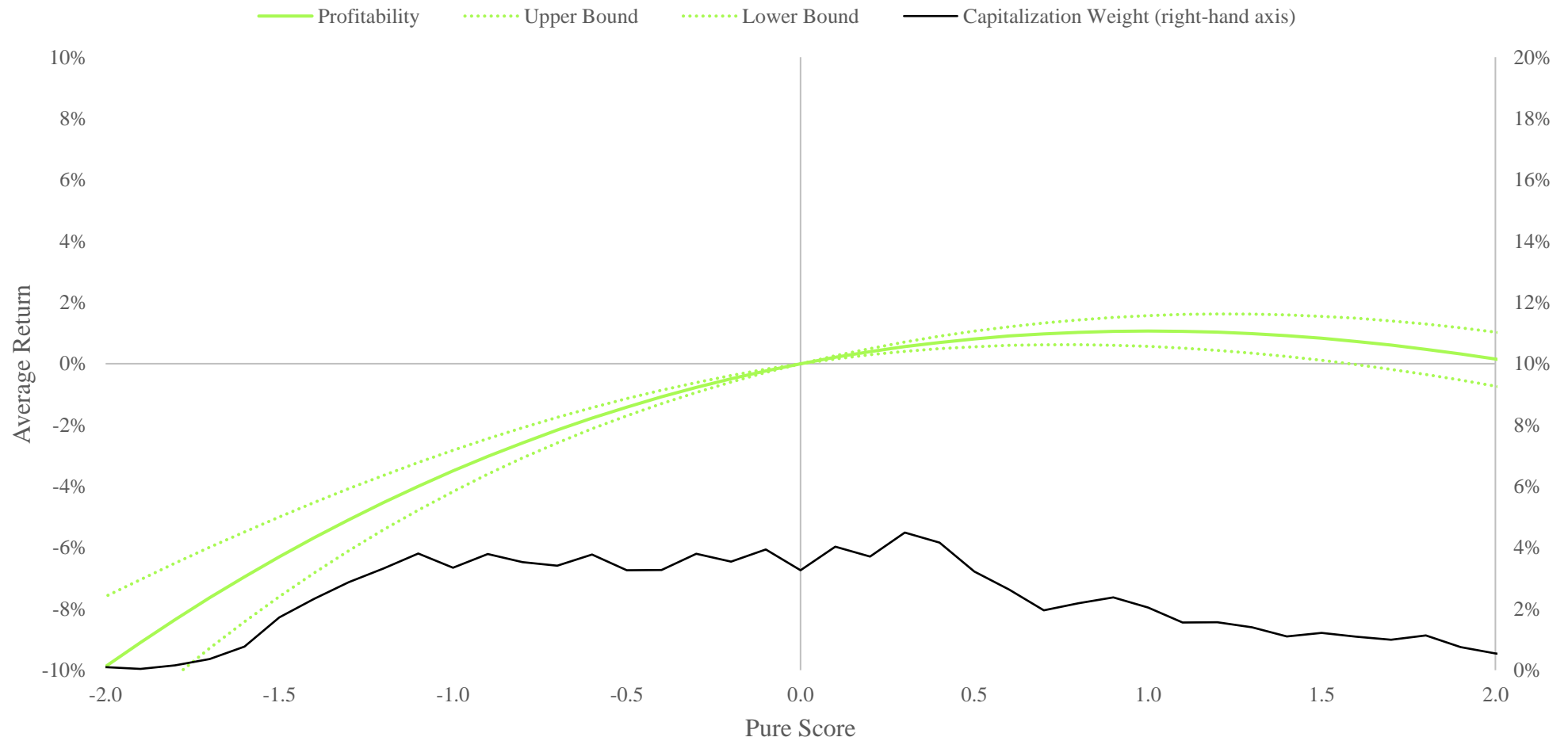


Note: Price Momentum is the logged version of Carhart's prior annual return less prior month return



Return to Profitability

Non-Linear Average Return to Pure Score 2004 to 2023

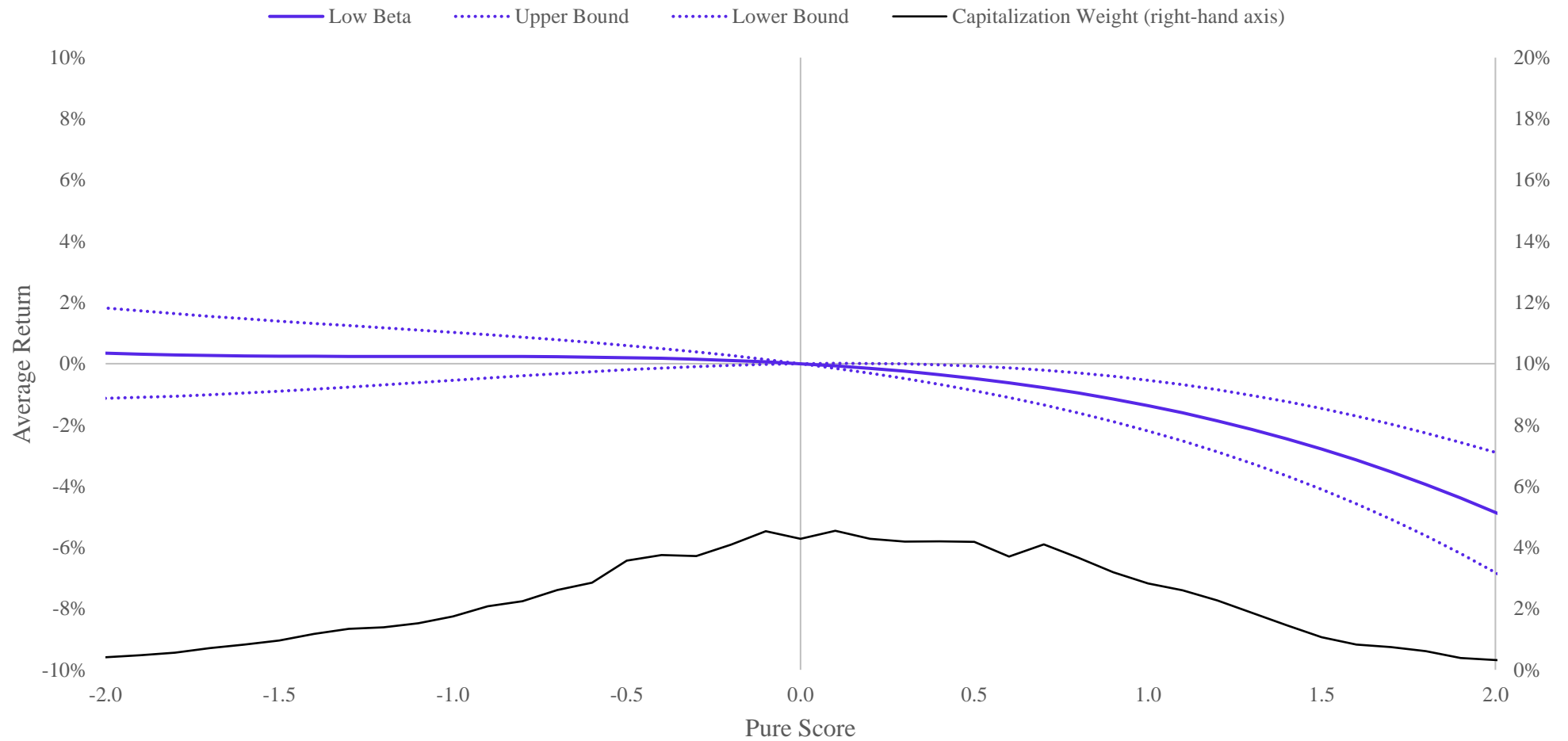


Note: Profitability is the prior year gross profit margin as specified by Novy-Marx



Return to Low Beta

Non-Linear Average Return to Pure Score 2004 to 2023

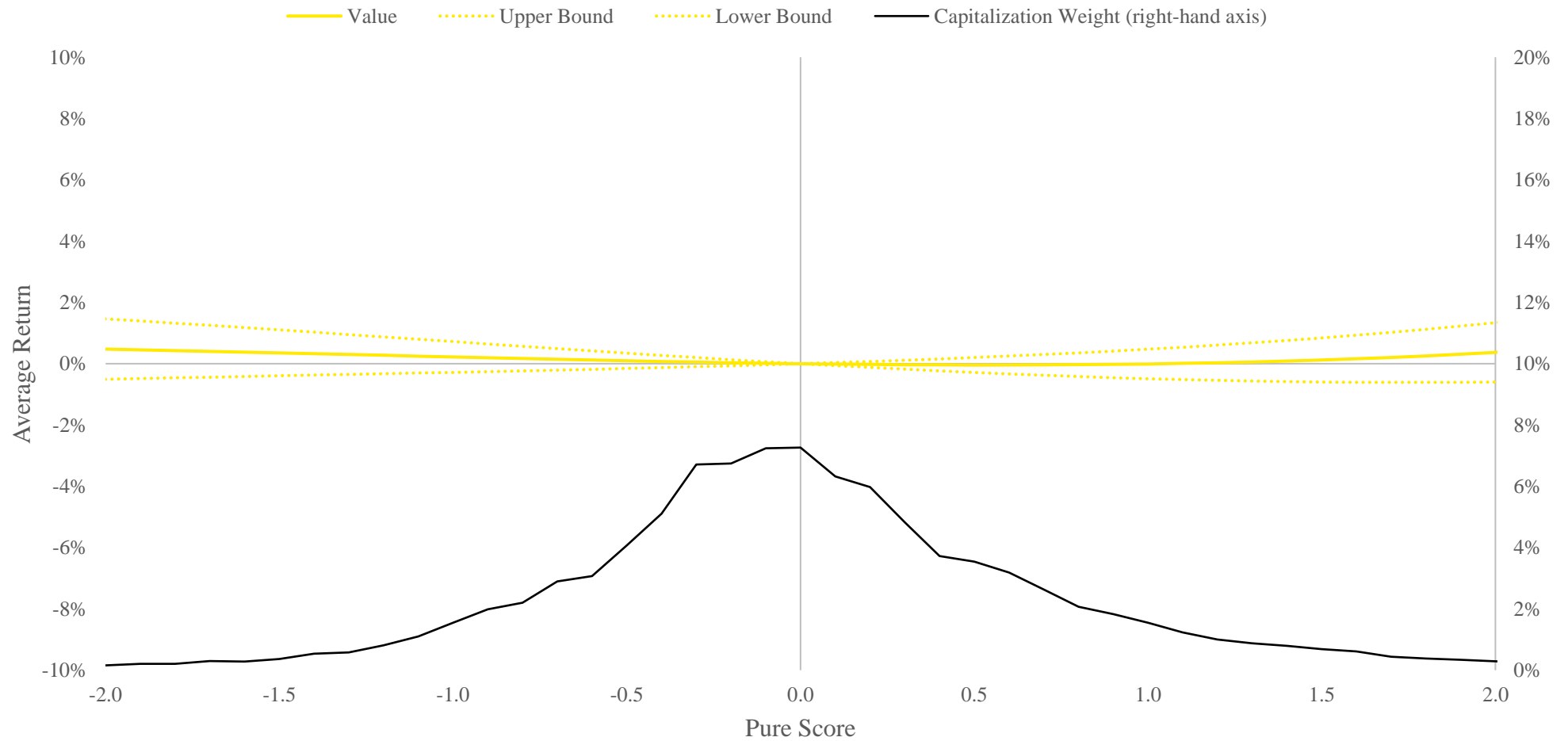


Note: Low Beta is one minus the trailing 36-month S&P 500 beta



Return to Value

Non-Linear Average Return to Pure Score 2004 to 2023

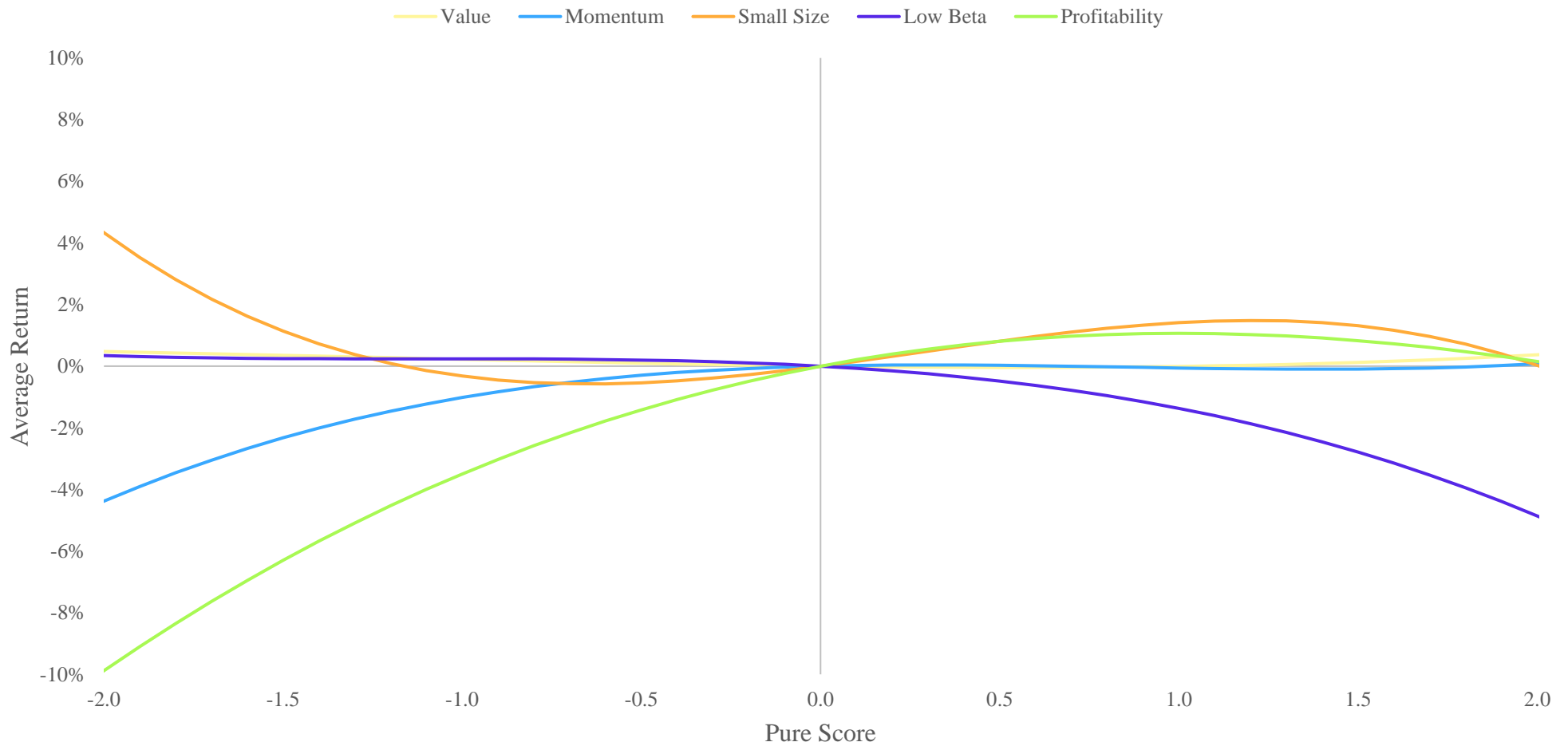


Note: Value is trailing annual earnings yield



Expected Return vs. Exposure

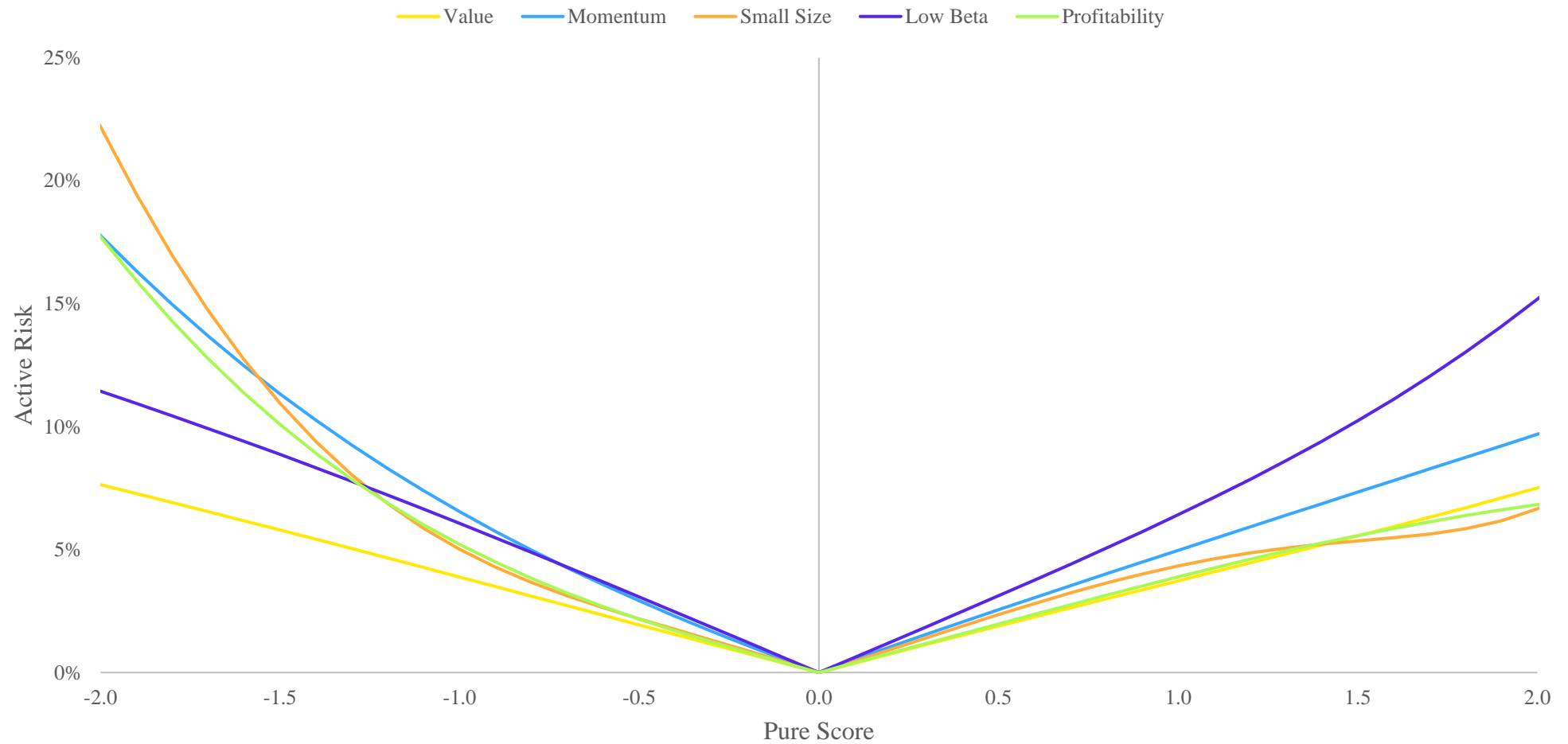
Average Active Return to Pure Score 2004 to 2023





Expected Tracking Error vs. Exposure

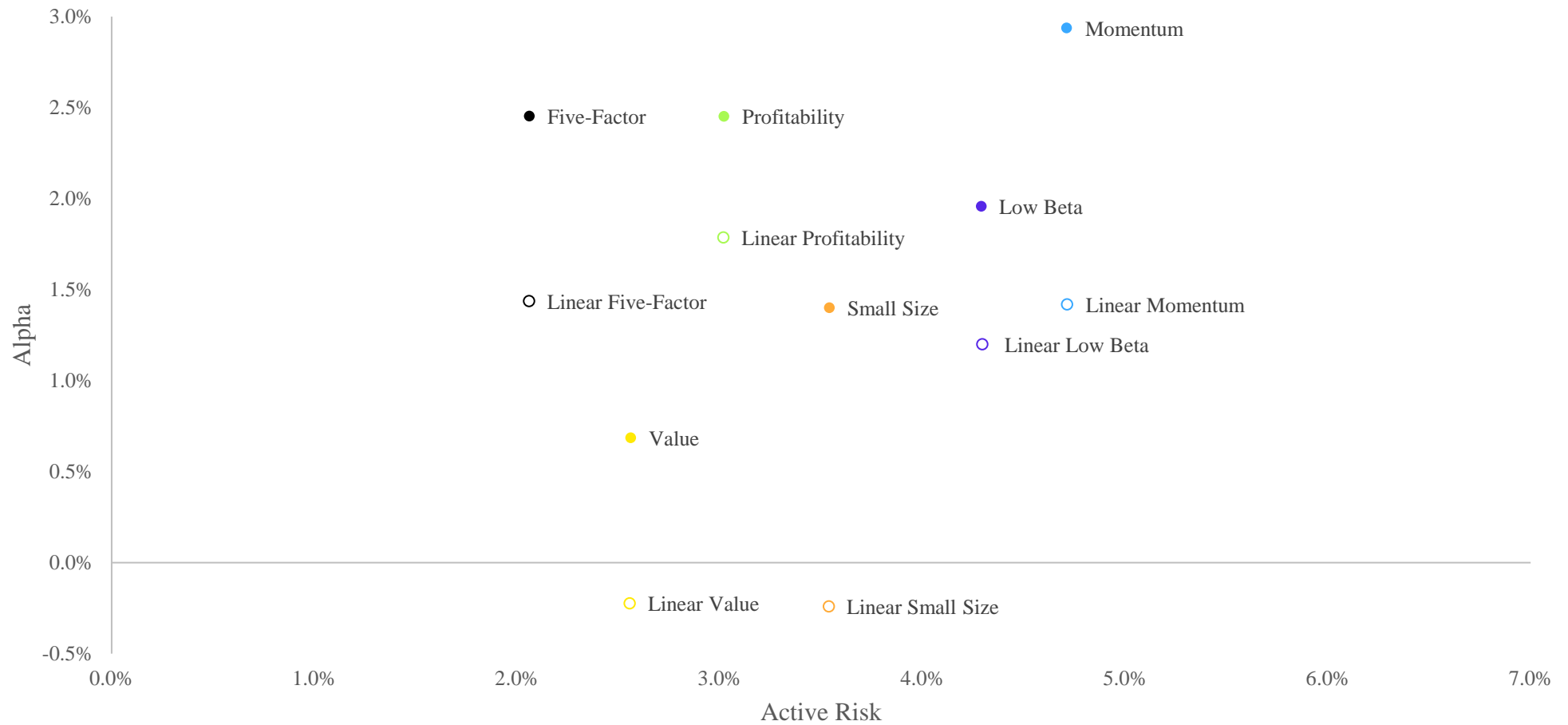
Active Risk to Pure Score 2004 to 2023





Worth the Effort?

Non-Linear Factor Portfolios in Active Return Space
Average Monthly Returns from 2003 to 2024





Historically Less Pronounced

Non-Linear Factor Portfolios in Active Return Space
Average Monthly Returns from 1984 to 2003





Near Linear

Non-Linear Factor Portfolios in Active Return Space
Average Monthly Returns from 1964 to 1983





Takeaways

- Average exposure can be misleading when exposures are non-linearly related to returns
- Be aware of the distribution of active exposure vs. the simple average exposure to factors
 - Contribution from “under-weights”
 - Contribution from “over-weights”
 - Is contribution from small active weight in tails or large active weight to stocks with moderate exposure?
- The risk model will underestimate risk if the non-linearity is large
- The non-linear component of factor return can be as large as the linear